Due Time :
Nov. 20th, 2010

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1) A block diagram shown in figure 1 represent a linear time invariant (LTI) dynamic system,
a) Derive the state space equations corresponds to the block diagram.
b) Simulate the system in Matlab/SIMULINK and test it fot a unit step input. Is the system a stable system or an unstable one?

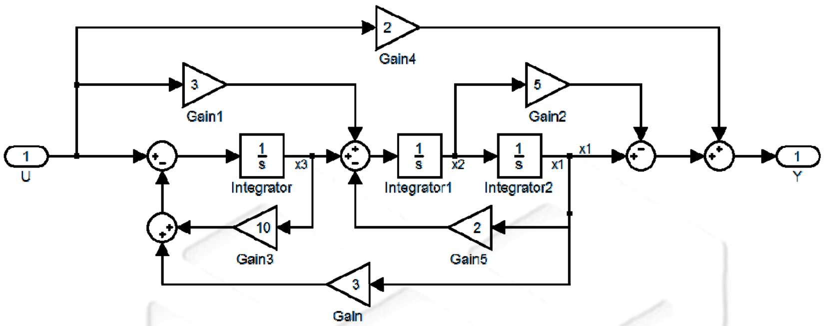

Fig.1 Block diagram of a LTI system

2) Derive the state space equation for the inverted pendulum shown in figure 2. Consider θ and x as the only outputs of system. Draw a block diagram for state space representation as figure 1.
(Hint: The system contains a continues mass pendulum bar with mass m linked to the base cart with mass M. The pendulum is able to rotate about the hinge while the cart is free to move horizontally. The system is nonlinear by its nature; It's needed to linearize the system around the top working equilibrium point.)

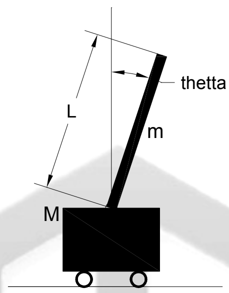

Fig.2 Inverted pendulum with continues mass bar

3) A train is considered as shown in figure 3, the train has a traction and 2 passenger wagons. Traction has a mass of m_L while wagons mass are m_W. The wagon connections are modeled with a spring and damper. The friction between wagon wheels and railway is modeled as a damper. The inputs to the model are F_t as the motive force and F_b as braking force. Derive a state space representation for the system with the inputs of F_t and F_b and outputs of $\dot{x}_1$, $\dot{x}_2$ and $\dot{x}_3$.

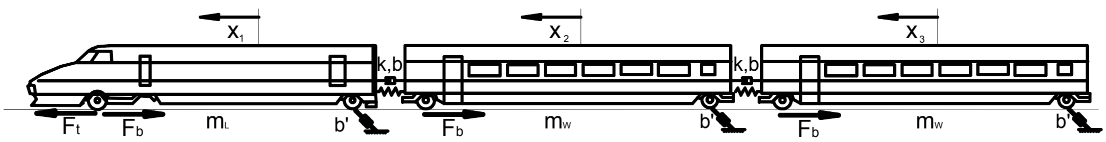

Fig.3 The train

4) An engine is illustrated in figure 4, the inputs to system are ambient pressure P_0 and external load on engine L . The engine speed is only output of system.

a) Determine the states of system.

b) What's the order of system?

c) Derive a state space equation for system using following physical equations

d) draw the block diagram for the state space system (just like figure 1)

$$\dot{m}_1 = c_1 (P_0 - P_i)$$

$$\dot{m}_2 = c_2 \omega$$

$$\dot{P}_i = c_3 (\dot{m}_1 - \dot{m}_2)$$

$$T = c_4 \dot{m}_2$$

$$\dot{\omega} = c_5 (T - L)$$

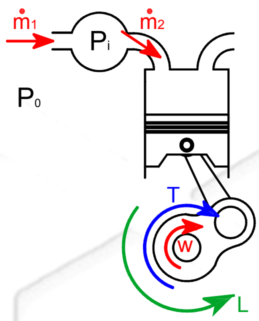

Fig.4 Internal combustion engine

5) A dynamic system is modeled with the following state space equation:

$$\dot{X} = \begin{bmatrix} -0.125 & -0.5 \\ 1 & 0 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 0.3 & 0.5 \\ 1 & 0.7 \end{bmatrix} X$$

Develop a new state space system with the current output values (Y) as its states.

Good luck